Section A

Question 1 (20 marks)

(14 marks) A bridge is required to span continuously over four supports. The spans are 10 m, 30 m and 10 m with flexural rigidity of EI, $2EI$ and $1.5EI$ respectively (see Figure 1). The bridge consists of two plate girders each weighing 4 kN/m and a deck weighing 10 kN/m. The design live load is 30 kN/m. Calculate the bending moments induced, highlighting the values at supports and at the centre of the spans, when the live load occupies only the right-hand and central spans. The partial safety factors to be applied are 1.6 for live load and 1.4 for dead load on all spans.

(6 marks) Calculate the four vertical reactions in the structure.

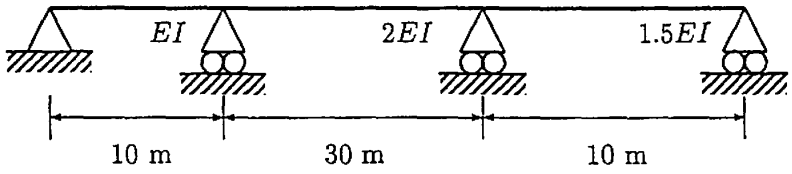

Figure 1: Continuous three-span bridge (not to scale)

Question 2 (20 marks)

(12 marks) A two-axle vehicle is to be considered in the design of a two-span continuous bridge. The spans are made of beams with the same sectional properties. The vehicle might be placed at any position on the bridge. Estimate the worst mid-span sagging moments for the vehicle given the data below. You need apply no load factors. Appropriate influence lines are provided on page 3.

Axle loads: 80 kN
Axle spacing: 6 m
Spans: 40 m and 20 m

(8 marks) Identify and carefully describe the purpose of the main structural elements of a typical plate girder to ensure its safe performance under ULS conditions.

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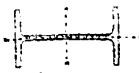

Designation	Mass per metre	Depth of Section h	Width of Section b	Thickness Web s	Thickness Flange t	Root Radius r	Depth between fillets d	Ratios for Local Buckling: Flange b/2t	Ratios for Local Buckling: Web d/s	Second Moment of Area: Axis x-x	Second Moment of Area: Axis y-y
	kg/m	mm	mm	mm	mm	mm	mm			cm⁴	cm⁴
406 x 178 x 74	74.2	412.8	179.5	9.5	16.0	10.2	360.4	5.61	37.9	27310	1545
406 x 178 x 67	67.1	408.4	178.8	8.8	14.3	10.2	360.4	6.25	41.0	24330	1355
406 x 178 x 60	60.1	406.4	177.9	7.9	12.8	10.2	360.4	6.95	45.6	21600	1203
406 x 178 x 54	54.1	402.6	177.7	7.7	10.9	10.2	360.4	8.15	46.8	16720	1021
406 x 140 x 46	46.0	403.2	142.2	6.8	11.2	10.2	360.4	6.35	53.0	15690	538
406 x 140 x 39	39.0	398.0	141.8	6.4	8.6	10.2	360.4	8.24	56.3	12510	410
356 x 171 x 67	67.1	363.4	173.2	9.1	15.7	10.2	311.6	5.52	34.2	19460	1352
356 x 171 x 57	57.0	358.0	172.2	8.1	13.0	10.2	311.6	6.62	38.5	16040	1103
356 x 171 x 51	51.0	355.6	171.5	7.4	11.5	10.2	311.6	7.46	42.1	14140	968
356 x 171 x 45	45.0	351.4	171.1	7.0	9.7	10.2	311.6	8.82	44.5	12070	811
356 x 127 x 39	39.1	353.4	126.0	6.6	10.7	10.2	311.6	5.89	47.2	10170	358
356 x 127 x 33	33.1	349.0	125.4	6.0	8.5	10.2	311.6	7.38	51.9	8249	280
305 x 165 x 54	54.0	310.4	166.9	7.9	13.7	8.9	265.2	6.09	33.6	11700	1063
305 x 165 x 46	46.1	306.6	165.7	6.7	11.8	8.9	265.2	7.02	39.6	9959	896
305 x 165 x 40	40.3	303.4	165.0	6.0	10.2	8.9	265.2	8.09	44.2	8503	764
305 x 127 x 48	48.1	311.0	125.3	9.0	14.0	8.9	265.2	4.47	29.5	9575	481
305 x 127 x 42	41.9	307.2	124.3	8.0	12.1	8.9	265.2	5.14	33.2	8196	389
305 x 127 x 37	37.0	304.4	123.3	7.1	10.7	8.9	265.2	5.77	37.4	7171	336
305 x 102 x 33	32.8	312.7	102.4	6.6	10.8	7.6	275.9	4.74	41.8	6501	194
305 x 102 x 28	28.2	308.7	101.8	6.0	8.8	7.6	275.9	5.78	46.0	5356	155
305 x 102 x 25	24.8	305.1	101.6	5.8	7.0	7.6	279.3	7.26	47.6	4455	123
254 x 146 x 43	43.0	259.6	147.3	7.2	12.7	7.6	219.0	5.80	30.4	6544	677
254 x 146 x 37	37.0	256.0	146.4	6.3	10.9	7.6	219.0	6.72	34.8	5537	571
254 x 146 x 31	31.1	251.4	146.1	6.0	8.6	7.6	219.0	8.49	36.5	4413	448
254 x 102 x 28	28.3	260.4	102.2	6.3	10.0	7.6	225.2	5.11	35.7	4005	170
254 x 102 x 25	25.2	257.2	101.9	6.0	8.4	7.6	225.2	6.07	37.5	3415	149
254 x 102 x 22	22.0	254.0	101.6	5.7	6.8	7.6	225.2	7.47	39.5	2841	119
203 x 133 x 30	30.0	206.8	133.9	6.4	9.6	7.6	172.4	6.97	26.9	2896	385
203 x 133 x 25	25.1	203.2	133.2	5.7	7.8	7.6	172.4	8.54	30.2	2340	308
203 x 102 x 23	23.1	203.2	101.8	5.4	9.3	7.6	169.4	5.47	31.4	2105	164
178 x 102 x 19	19.0	177.8	101.2	4.8	7.9	7.6	146.8	6.41	30.6	1356	137
152 x 89 x 16	16.0	152.4	88.7	4.5	7.7	7.6	121.6	5.76	27.1	834	69.8
127 x 76 x 13	13.0	127.0	76.0	4.0	7.6	7.6	98.6	5.00	24.1	473	55.7

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Designation	Radius of Gyration: Axis x-x	Radius of Gyration: Axis y-y	Elastic Modulus: Axis x-x	Elastic Modulus: Axis y-y	Plastic Modulus: Axis x-x	Plastic Modulus: Axis y-y	Buckling Parameter u	Torsional Index x	Warping Constant H	Torsional Constant J	Area of Section	Mass per metre	Designation
	cm	cm	cm³	cm³	cm³	cm³			dm⁶	cm⁴	cm²	kg/m	
406 x 178 x 74	17.0	4.04	1323	172	1501	267	0.882	27.6	0.608	62.8	94.5	74.2	406 x 178 x 74
406 x 178 x 67	16.9	3.99	1159	153	1346	237	0.880	30.5	0.533	46.1	85.5	67.1	406 x 178 x 67
406 x 178 x 60	16.8	3.97	1063	135	1199	209	0.880	33.6	0.466	33.3	76.5	60.1	406 x 178 x 60
406 x 178 x 54	16.5	3.85	930	115	1065	178	0.871	36.3	0.392	23.1	69.0	54.1	406 x 178 x 54
406 x 140 x 46	16.4	3.03	778	75.7	858	118	0.871	38.9	0.207	19.0	58.6	46.0	406 x 140 x 46
406 x 140 x 39	15.9	2.87	629	57.8	724	90.8	0.858	47.5	0.155	10.7	49.7	39.0	406 x 140 x 39
356 x 171 x 67	15.1	3.99	1071	167	1211	243	0.886	24.4	0.412	55.7	85.5	67.1	356 x 171 x 67
356 x 171 x 57	14.9	3.91	896	129	1010	193	0.882	28.8	0.330	33.4	72.6	57.0	356 x 171 x 57
356 x 171 x 51	14.8	3.88	796	113	896	174	0.881	32.1	0.288	23.8	64.9	51.0	356 x 171 x 51
356 x 171 x 45	14.5	3.76	687	94.8	775	147	0.874	36.8	0.237	15.8	57.3	45.0	356 x 171 x 45
356 x 127 x 39	14.3	2.68	576	56.8	659	89.1	0.871	35.2	0.105	15.1	49.8	39.1	356 x 127 x 39
356 x 127 x 33	14.0	2.58	473	44.7	543	70.3	0.863	42.2	0.0812	8.76	42.1	33.1	356 x 127 x 33
305 x 165 x 54	13.0	3.93	754	127	846	159	0.889	23.6	0.234	34.8	68.8	54.0	305 x 165 x 54
305 x 165 x 46	13.0	3.90	646	103	720	166	0.891	27.1	0.195	22.2	58.7	46.1	305 x 165 x 46
305 x 165 x 40	12.9	3.86	560	92.6	623	142	0.889	31.0	0.164	14.7	51.3	40.3	305 x 165 x 40
305 x 127 x 48	12.5	2.74	616	73.6	711	116	0.873	22.3	0.102	31.8	61.2	48.1	305 x 127 x 48
305 x 127 x 42	12.4	2.70	534	62.6	614	93.4	0.872	26.5	0.0846	21.1	53.4	41.9	305 x 127 x 42
305 x 127 x 37	12.3	2.67	471	54.5	539	85.4	0.872	29.7	0.0725	14.6	47.2	37.0	305 x 127 x 37
305 x 102 x 33	12.5	2.15	416	37.9	480	60.0	0.866	31.8	0.0442	12.2	41.8	32.8	305 x 102 x 33
305 x 102 x 28	12.2	2.08	348	30.5	403	48.5	0.859	37.4	0.0340	7.40	35.9	28.2	305 x 102 x 28
305 x 102 x 25	11.9	1.97	292	24.2	342	38.6	0.846	43.4	0.0273	4.77	31.6	24.8	305 x 102 x 25
254 x 146 x 43	10.9	3.52	504	92.0	566	141	0.891	21.2	0.103	23.9	54.8	43.0	254 x 146 x 43
254 x 146 x 37	10.8	3.48	433	78.0	483	119	0.890	24.3	0.0857	15.3	47.2	37.0	254 x 146 x 37
254 x 146 x 31	10.5	3.36	351	61.3	393	94.1	0.880	29.6	0.0660	8.56	39.7	31.1	254 x 146 x 31
254 x 102 x 28	10.5	2.22	308	34.9	353	54.6	0.874	27.5	0.0360	9.67	36.1	28.3	254 x 102 x 28
254 x 102 x 25	10.3	2.15	266	29.2	308	46.0	0.866	31.5	0.0293	6.42	32.0	25.2	254 x 102 x 25
254 x 102 x 22	10.1	2.06	224	23.5	259	37.3	0.856	36.4	0.0182	4.15	28.0	22.0	254 x 102 x 22
203 x 133 x 30	8.71	3.17	280	57.5	314	89.2	0.891	21.5	0.0374	10.3	38.2	30.0	203 x 133 x 30
203 x 133 x 25	8.56	3.10	230	48.2	255	70.9	0.877	25.6	0.0294	5.96	32.0	25.1	203 x 133 x 25
203 x 102 x 23	8.46	2.35	207	32.2	234	49.8	0.883	22.5	0.0154	7.02	29.4	23.1	203 x 102 x 23
178 x 102 x 19	7.48	2.37	153	27.0	171	41.6	0.858	22.6	0.00987	4.41	24.3	19.0	178 x 102 x 19
152 x 89 x 16	6.41	2.10	109	20.2	123	31.2	0.850	19.6	0.00470	3.58	20.3	16.0	152 x 89 x 16
127 x 76 x 13	5.35	1.84	74.6	14.7	84.2	22.6	0.835	18.3	0.00199	2.85	16.5	13.0	127 x 76 x 13

Table 2: Section table for Question 3.

05

Section B

Question 4 (20 marks)

(a) (6 marks) Explain the significance of the Bernoulli hypothesis that plane sections remain plane in the context of reinforced concrete beams.

(b) For the beam section in Figure 3,

i. (6 marks) determine the flexural rigidity for SLS purposes and

ii. (8 marks) determine the ratio of moment and curvature at ULS corresponding to a flexural collapse, exclude material partial safety factors in this question.

Assume the following data: $E_c = 30\text{kN/mm}^2$; $f_{ck} = 40\text{N/mm}^2$; $E_s = 200\text{kN/mm}^2$; $f_{yk} = 460\text{N/mm}^2$.

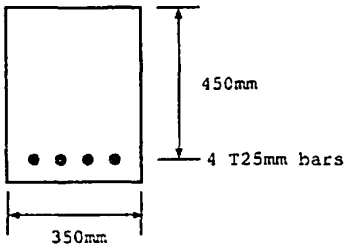

Figure 3: Cross-section for Question 4. Not to scale.

Table A.1 Sectional areas of groups of bars (mm^2)

Bar size (mm)	*Number of bars*									
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
6	28.3	56.6	84.9	113	142	170	198	226	255	283
8	50.3	101	151	201	252	302	352	402	453	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	566	679	792	905	1 020	1 130
16	201	402	603	804	1010	1210	1410	1610	1 810	2 010
20	314	628	943	1260	1570	1890	2200	2510	2 830	3 140
25	491	982	1470	1960	2450	2950	3440	3 930	4 420	4 910
32	804	1610	2410	3220	4020	4830	5630	6 430	7 240	8 040
40	1260	2510	3770	5030	6280	7540	8800	10 100	11 300	12 600

Table 3: Table of bar areas